GGSIPU mathmatics 2013

1. Concentric circles of radii 1,2,3,……,100 cm are drawn. The interior of the smallest circle is coloured red and the angular regions are coloured red and the angular regions are coloured alternately green and red, so that no two adjacent regions are of the same colour. The total area of the green region in sq.cm is equal to

a 1000 π b 5050 π

c 4950 π d 5151 π

2. The value of a for which the quadratic equation

$3x^2 + 2a^2 +1 x + a^2 -3a+2 = 0$

Possesses roots of opposite signs lies in

a $-\infty, 1$
b $(-\infty\ 0)$
c $(1,2)$
d $\left(\frac{3}{2}, 2\right)$

3. If $2z_1 -3z_2-z_3 = 0$, then z_1,z_2 and z_3 are represented by

a three of a triangle

b three collinear points

c three vertices of a rhombus

d None of the above

4. The term independent of x,in the expension of $\left(1+\frac{1}{x}+x+x^2\right)^4$ is

a 35 b 30 c 32 d 31

5. The number of six-digit numbers which have sum of their digits as an odd integer, is

a 45000 b 450000

c 97000 d 970000

6. Consider the Δ AOB in the x,y-plane where A $\equiv$ 1,0,0, B $\equiv$0,0,0. The new position of O, when triangle is rotated about side AB by 90^0 can be

a $\left(\frac{4}{5}, \frac{3}{5}, \frac{2}{\sqrt{5}}\right)$

b $\left(\frac{-3}{5}, \frac{\sqrt{2}}{5}, \frac{2}{\sqrt{5}}\right)$

c $\left(\frac{4}{5}, \frac{2}{5}, \frac{2}{\sqrt{5}}\right)$

d $\left(\frac{4}{5}, \frac{2}{5}, \frac{1}{\sqrt{5}}\right)$

7. Number of planes which are at a given perpendicular distance from a given point and passing through a given point is

a 0 b 2 c 4 d infinite

8. If A and B are two independent events, then which of the following is not equal to any of the remaining?

a PA' $\cap$ B' –P A $\cap$ B

b PA' +PB' – 1

c PB – PA'

d PB' – PA

9. In $u_n = 2\cos n\theta$ and $u_1 u_n - u_{n-1}$ is equal to

a u_{n-2} b u_{n+1}

c 0 d1) None of these

10. If $\frac{1}{\sqrt{2}} < x < 1$, then $\cos^{-1} x + \cos^{-1}\left(\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right)$ is equal to

a 2 cosx $^{-1}$ b 2 cos $^{-1}$ x

c $\frac{\pi}{4}$ d 0

11. The number of values of θ satisfying $4\cos\theta + 3\sin\theta = 5$ as well as $3\cos\theta + 4\sin\theta = 5$ is

a 1 b 2

c 0 d None of these

12. A kite is flying with the string inclined at 75^0 to the horizon. If the length of the string is 25 m, then height of the kite is

a $\left(\frac{25}{2}\right)(\sqrt{3}$ -1 ^{2} b $\left(\frac{25}{2}\right)(\sqrt{3}+1\ \sqrt{2}$

c $\left(\frac{25}{2}\right)(\sqrt{3}+1$ ^{2} d $\left(\frac{25}{2}\right)(\sqrt{6}+\sqrt{2}$

13. The ends of a quadrant of a circle have the coordinates 1,3 and 3,1. Then, the centre of circle is

a 2,2 b 1,1

c 4,4 d 2,6

14. If the latus rectum of the parabola $2x^2$-ky+2 = 0 be 2, then the vertex is

a $\left(0,\frac{3}{4}\right)$ b $\left(0,\frac{3}{2}\right)$

c $\left(\frac{3}{4},0\right)$ d 0,0

15. If f: 3,4 → 0,1 is defined by f(x = x – [x], where [x] denotes the greatest integer function, then f'x is

a $\frac{1}{x-[x]}$ b [x] – x

(c) x-3 d x+3

16. If f(x = cos $^{-1}\frac{x-x^{-1}}{x+x^{-1}}$ then f' -2 is

a $\frac{2}{5}$ b $\frac{-2}{5}$

c $-\frac{1}{5}$ d None of these

17. Let f(x be an even function in R. If f(x is monotonic increasing in [2,6], then

a f3 > f(-5 b f -2 < f2

c:) f(-2> f2 d f(-3<f(5)

18. If $\int_{a-n}^{n} e^{x\ a-x)}$dx = λ, then the value of $\int_{a}^{n} xe^{x\ a-x)}$dx, a ≠ 2n, is

a $\frac{a\lambda}{2}$ b a λ

c 2a λ d None of these

19. If I = $\int_{1/\pi}^{\pi} \frac{1}{x}$. Sin $\left(x-\frac{1}{x}\right)$ dx, then I is equal to

a 0 b π c $\pi-\frac{1}{\pi}$ d $\pi+\frac{1}{\pi}$

20. The number of sides of the quadrilateral whose joint equation is $x^2y^2 + 1 = x^2 + y^2$, and which are touched by the circles $x^2 + y^2 = 2x$ is

a 4 b 3 c 2 d 1

21. If f(x+2 = $\frac{1}{2}$ {fx+1 + $\frac{4}{f'x}$} and f(x > 0, for all x $\in$ R, then $\lim_{x\to\infty} f(x)$ is

a 1 b 2 c -2 d 0

22. Let f(x be a continuous function whose range is [2,6,5]. If hx = $\left[\frac{\cos x + f(x)}{\lambda}\right]$, $\lambda \in$ N be continuous, where [.] denotes the greatest integer function, then the least value of λ is

a 6 b 7

(c) :_ d ,_None of these

23. $\int \frac{3+2cosx}{2+3\cos x)^2}$. dx is equal to

a $\left(\frac{\sin x}{2+3\cos x}\right)$ + c

b $\left(\frac{\sin x}{2+3\sin x}\right)$ + c

c Both a and b

d None of the above

24. Differential equation of the family of circles touching the line y = 2 at 0,2 is

a x ^{2} + y -2 ^{2} + $\frac{dy}{dx}$y-2)=)

b x ^{2} + y -2 $\left(2 - 2x\frac{dx}{dy} - y\right)$ = 0

c x ^{2} + y -2 ^{2} + $\left(\frac{dx}{dy} + y - 2\right)$ (y-2 = 0

d None of the above

25. If a,b and c are non -zero real numbers and az^2+bz+c+I = 0 has purely imaginary roots,then a is equal to

a bc b b ^{2}c c -b^{2}c d $\frac{1}{2}$b^{2}c

26. If a,b and c are three mutually orthogonal unit vectors, then the triple product [a+b+c a+b b+c] is equal to

a 0 b 1 or -1 c 1 d 3

27. y^2 = 4x is a curve and P,Q and r are three points on it, where P = 1,2, Q = $\left(\frac{1}{4}, 1\right)$ and the tangent to the curve at R is parallel to the chord PQ of the curve, then coordinates of R are

a $\left(\frac{5}{8}, \sqrt{\frac{5}{2}}\right)$ (b) $\left(\frac{9}{16}, \frac{3}{2}\right)$

c $\left(\frac{5}{8}, -\sqrt{\frac{5}{2}}\right)$ d $\left(\frac{9}{16}, \frac{-3}{2}\right)$

28. A batsman can score 0,1,2,3,4 or 6 runs from a ball. The number of different sequences in which he can score exactly +30 runs in an over of six balls, is

a 4 b 72 c 56 d 7

29. If x f(y = fx f(y ^{2}, $\forall$ x, y $\in$ R and f($\sqrt{3}$ + f $\sqrt{5}$ = 4, then f' $\sqrt{3}$ is equal to

a 1 b $\sqrt{3}$ c $-\sqrt{3}$ d 1

30. The number of solutions for the equation $2\sin^{-1}\sqrt{(x^2-x+1} + \cos^{-1}\sqrt{(x^2-x} = \frac{3\pi}{2}$ is

a 1 b 2 c 3 d Infinite

31. The number of solutions of the equation $\int_{-2}^{x} /\cos x/\, dx = 0,\ 0<x<\frac{\pi}{2}$, is

a 0 b 1 c 2 d 4

32. A person standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60^0, when he retires 40m from the bank he finds the angle to 30^0. The breadth of river is

a 40 m b 60 m

c 20 m d 30 m

33. Two circles $x^2+y^2-2kx = 0$ and $x^2+y^2-4x-8y+16 = 0$ touch each other externally. Then , k is

a 4 b 1

b 2 d -4

34. If the line ax+by = 2 is a normal to the circle $x^2+y^2-4x-4y = 0$ and a tangent to the circle $x^2+y^2 = 1$, then

a a = $\frac{1}{2}$, b = $\frac{1}{2}$

b a = $\frac{1+\sqrt{7}}{2}$, b = $\frac{1-\sqrt{7}}{2}$

c a = $\frac{1}{4}$, b = $\frac{3}{4}$

d a = 1, b = $\sqrt{3}$

35. The graph of the curve $x^2+y^2-2xy-8x-8y+32 = 0$ falls wholly in the

a first quadrant b second quadrant

c third quadr ant d Noone o:hese

36. The number of solutions [cos x] + $|sin\ x|$ = 1 in $\pi \leq x < 3\pi$ is

a 3 b 4 c 2 d 1

37. The slope of the tangent to the curve tan y = $\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$ at x = $\frac{1}{2}$ is

a $\frac{1}{\sqrt{3}}$ b $\sqrt{3}$

c 1 d $\frac{1}{2}$

38. If the real val;ued function f(x = x ^{3}+3a ^{2}-1 x + 1 be invertible, then set of possible real values of a is

a - ∞, - 1 ∪ 1, ∞ b -1,1

c [-1,1] (d) (- ∞, - 1] ∪ 1, + ∞

39. The value of $\int_0^{\pi/4} \frac{\sec x}{\sec x+\tan x)^2}$. dx is

a 1+ $\sqrt{2}$ b -11+ $\sqrt{2}$

c -$\sqrt{2}$ d None of these

40. The combined equation of straight lines that can be obtained by reflecting the lines y = [x-2] in the y-axis is

a y $^2+x^2+4x+4 = 0$

b y $^2+x^2-4x+4 = 0$

c y $^2-x^2+4x-4 = 0$

(d) $y^2-x^2-4x-4 = 0$

41. $\lim_{x\to 0}\left\{(1+x^{\frac{2}{x}}\right\}$, where {.} denotes the fractional part of x,is equal to

a e $^2 - 7$ b e $^2 - 8$

c) $e^2 - 6$ (d) None of tese

42. fx = $\begin{cases} e^{-1/x^2} & , x > 0 \\ 0 & , x \le 0 \end{cases}$, then f(x is

a differential at x =0

b continuous but not differentiable at x =0

c discontinuous a t x =0

d)None of the above

43. $\int \frac{1}{x^2 \, x^4+1)^{3/4}}$ dx is equal to

a $\left(1+\frac{1}{x^4}\right)^{1/4}$ + C b $\left(x^4+1\right)^{1/4}$ + C

c $\left(1-\frac{1}{x^4}\right)^{1/4}$ + C d $-\left(1+\frac{1}{x^4}\right)^{1/4}$ + C

44. The solution of the differential equation (1+ x^2y^2 y dx + x^2y^2 - 1 xdy = 0 is

a xy = log $\frac{x}{y}$ + C b xy =2 log $\frac{y}{x}$ + C

c x^2y^2 = 2 log $\frac{y}{x}$ + C d None of these

45. Equation of chord of contact of pair of tangents, drawn to ellipse $4x^2+9y^2 = 36$ from the point m,n, where m.n = m+n, m,n being non -zero positive integers,is

a 2x+9y = 18 b 2x+2y = 1

c 4x+9y = 18 d None of these

46. The equation to the hyperbola of given transverse axis whose vertex bisects the distance between the centre and focus, is given by

a $3x^2-y^2 = 3a^2$ b $x^2-3y^2 = a^2$

c $x^2-y^2 = 3a^2$ d None of theese

47. The plane ax-by-cz = d will contain the line $\frac{x-a}{a} = \frac{y+3d}{b} = \frac{z-e}{c}$, provided

a b = [0,3 d] b a = [2d]

c c = [3d] d b = [-3d]

48. If z is a complex number lying in the first quadrant such that Rez + Imz = 3, then the maximum values of [Re z] ^{2} Im z is

a 1 (b) 2 3 (d) 4

49. If A = $\tan^{-1}\left(\frac{x\sqrt{3}}{2k-x}\right)$ and B = $\tan^{-1}\left(\frac{2x-k}{k\sqrt{3}}\right)$. Then, A – B is equal to

a $\frac{\pi}{2}$ b $\frac{\pi}{3}$

c $\frac{\pi}{6}$ d None of these

50. If in a $\triangle ABC$, $\angle B = \frac{2\pi}{3}$, then the cos A + cos C lies in

a [$[-\sqrt{3}, \sqrt{3}]$ (b) $(-\sqrt{3}, \sqrt{3})$

c $\left(\frac{3}{2}, \sqrt{3}\right]$ (d) $\left[\frac{3}{2}, \sqrt{3}\right]$